FUNCTIONS 2
STUDENT PACKET

SLOPE AND SLOPE-INTERCEPT FORM OF A LINE

Parent (or Guardian) signature ____________________________________

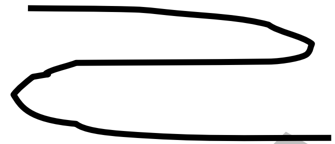
MY WORD BANK

Explain the mathematical meaning of each word or phrase, using pictures and examples when possible. (See section 1.5.) Key mathematical vocabulary is underlined throughout the packet.

parallel lines	slope-intercept form
slope of a line	*y*-intercept

THE ROPE PROBLEM

Follow your teacher's directions to explore layers, cuts, and pieces in this rope problem.
Use extra paper if needed.



SLOPE

We will explore the meaning of positive and negative slopes of lines. We will count to find distances and slopes of lines on a grid. Then we will use slope definitions to find slopes of lines in the coordinate plane.

GETTING STARTED

1. Circle all of the expressions below that are equivalent to $10 \div 2$.

$10 \div (-2)$ $\quad (-10) \div 2$ $\quad (-10) \div (-2)$ $\quad 20 \div 4$ $\quad (-20) \div (-4)$ $\quad 20 \div (-4)$ $\quad (-20) \div 4$

2. Circle all of the numbers below that are equivalent to $\frac{1}{2}$.

$\frac{-1}{-2}$ $\quad \frac{-1}{2}$ $\quad \frac{1}{-2}$ $\quad -\frac{1}{2}$ $\quad \frac{4}{8}$ $\quad -\frac{4}{8}$ $\quad \frac{4}{-8}$ $\quad \frac{-4}{8}$

3. Circle all of the numbers below that are equivalent to $-\frac{3}{4}$.

$\frac{-3}{4}$ $\quad \frac{3}{-4}$ $\quad \frac{-3}{-4}$ $\quad -\frac{3}{4}$ $\quad \frac{6}{8}$ $\quad -\frac{6}{8}$ $\quad \frac{-6}{-8}$ $\quad \frac{-6}{8}$

4. Circle all of the numbers below that are equivalent to $-\frac{5}{3}$.

$\frac{5}{3}$ $\quad \frac{-5}{-3}$ $\quad \frac{-5}{3}$ $\quad \frac{5}{-3}$ $\quad \frac{10}{6}$ $\quad -\frac{10}{6}$ $\quad \frac{10}{-6}$ $\quad \frac{-10}{-6}$

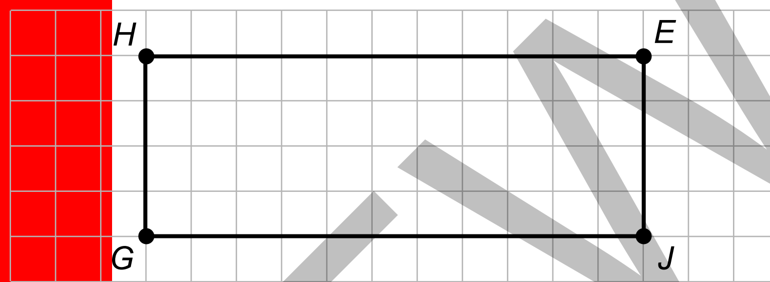
5. Circle all of the numbers below that are equivalent to $\frac{3}{1}$.

$\frac{-3}{-1}$ $\quad -\frac{3}{1}$ $\quad \frac{-3}{1}$ $\quad \frac{3}{-1}$ $\quad 3$ $\quad \frac{-6}{2}$ $\quad \frac{6}{-2}$ $\quad -\frac{9}{3}$ $\quad \frac{-12}{-4}$

GETTING STARTED
(Continued)

In this packet, each small square is one square unit.

6. Find each distance on the grid.



   a. From G to H:

   b. From G to J:

   c. From E to J:

7. Start with the given point and follow the directions to plot the next point.

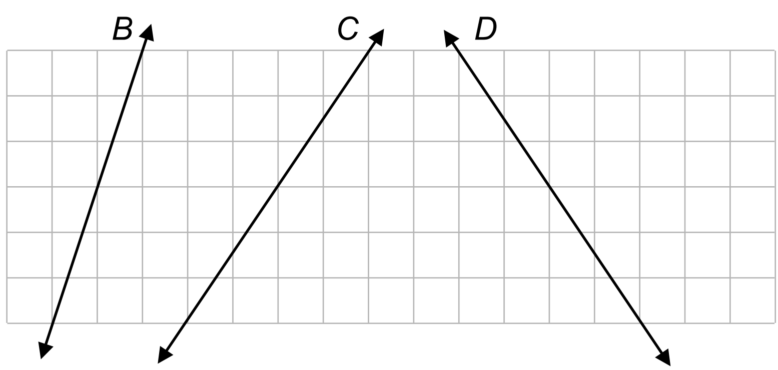
   a. Start at point A. Count 4 units down and 1 unit to the right. Plot point N. Draw $\overline{AN}$.

   b. Start at point O. Count 3 units up and 2 units to the right. Plot point F. Draw $\overline{OF}$.

   c. Start at point I. Count 1 unit down and 7 units to the left. Plot point T. Draw $\overline{IT}$.

   d. Start at point U. Count 2 units up and 4 units to the left. Plot point P. Draw $\overline{UP}$.

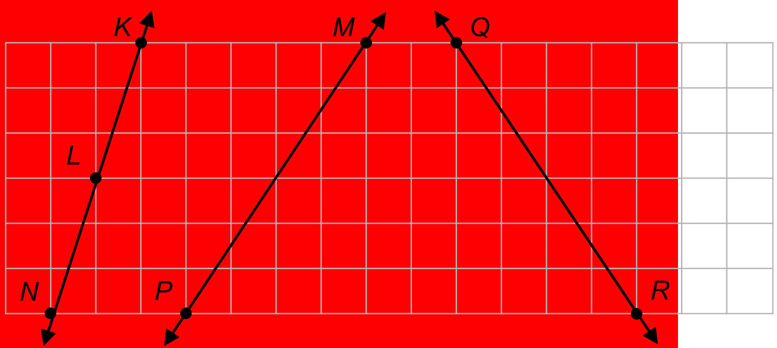
8. Which looks steeper to you?

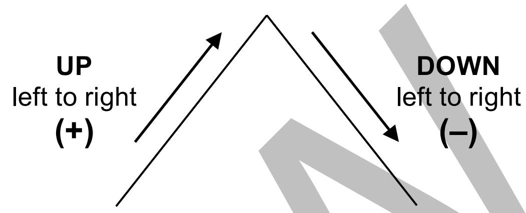


   a. Line B or line C?

   b. Line C or line D?

THE MEANING OF SLOPE





Just like reading words on a page, we read a graph from left to right.

Follow your teacher's directions for problems 1 – 8. Always note whether a line has a positive or negative slope (**+** or **–**) before finding the slope.

1. Explain the meaning of the <u>slope of a line</u>. See section 1.5 for help and record in My Word Bank.	
2. Slope of _____: **+** **–**	5. Slope of _____: **+** **–**
3. Slope of _____: **+** **–**	6. Slope of _____: **+** **–**
4. Slope of _____: **+** **–**	7. Slope of _____: **+** **–**
	8. Slope of _____: **+** **–**

9. What do you notice about the slope values for problems 2-4?

10. What do you notice about the slope values for problems 5-6?

11. Which value is greater, the slope of $\overline{NK}$ or the slope of $\overline{PM}$?

    Which line segment is steeper $\overline{NK}$ or $\overline{PM}$?

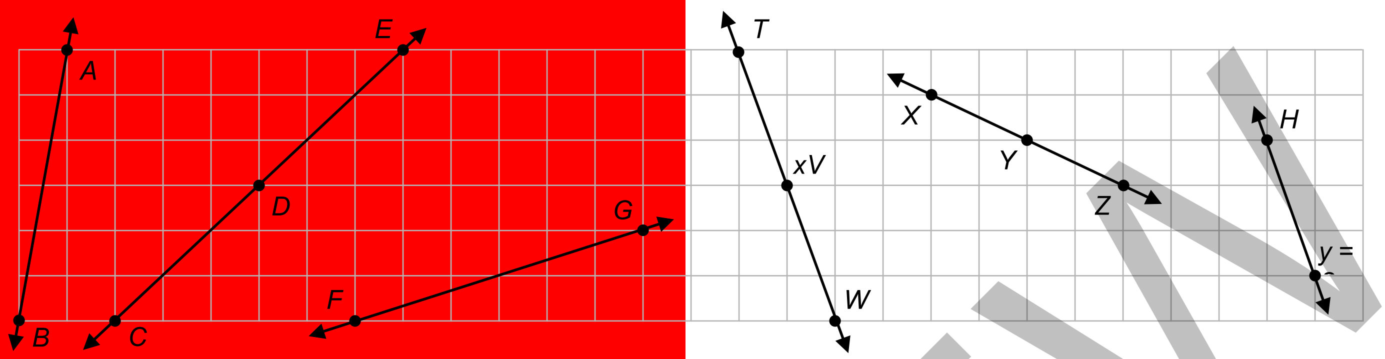
12. What do you notice about the slope values for problems 7-8?

13. How are the slope values in problems 5-6 related to the slope values in problems 7-8?

14. Which value is greater, the slope of $\overline{PM}$ or the slope of $\overline{RQ}$?

15. Which line is steeper $\overline{PM}$ or $\overline{RQ}$?

PRACTICE 1



Before counting, determine whether each slope is positive (**+**) or negative (**–**) and circle the appropriate symbol. Then find the slope of each line segment by counting.

1. Slope of $\overline{AB}$: **+** **–**	5. Slope of $\overline{TV}$: **+** **–**
2. Slope of $\overline{CD}$: **+** **–**	6. Slope of $\overline{WT}$: **+** **–**
3. Slope of $\overline{EC}$: **+** **–**	7. Slope of $\overline{XZ}$: **+** **–**
4. Slope of $\overline{FG}$: **+** **–**	8. Slope of $\overline{HJ}$: **+** **–**

9. What do you notice about the slopes of line segments lying on the same line?

10. Which segment from problems 1-8 must have the same slope as $\overline{XY}$?

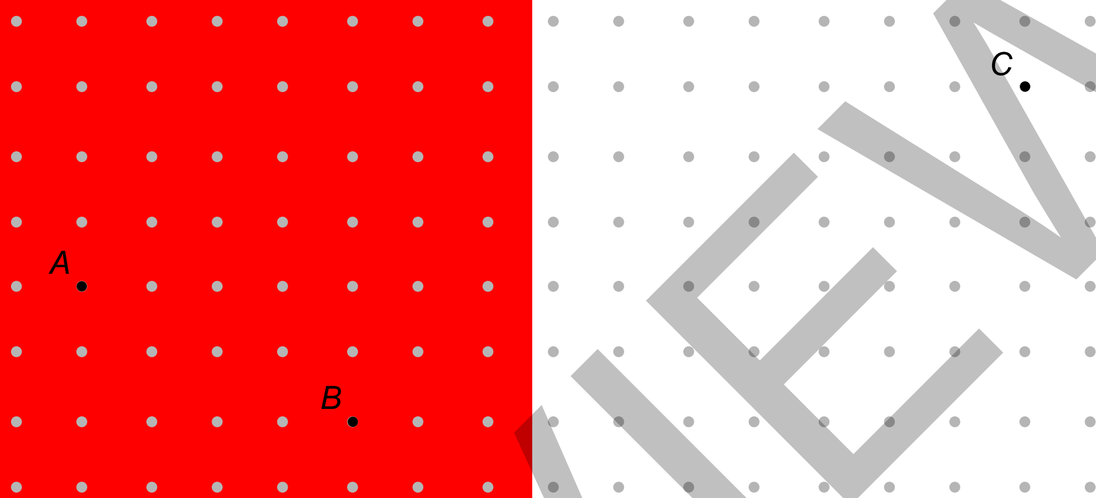
    Explain or show why this is correct.

11. Choose two different lines above that appear to be parallel.

    What do you notice about the slopes of these <u>parallel lines</u>?

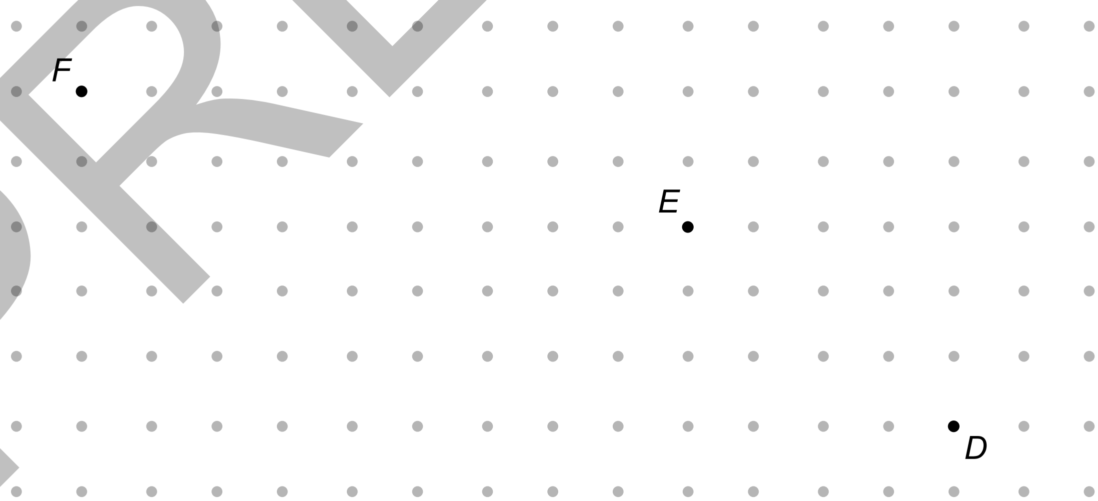
PRACTICE 2

1. For each point, count vertical and horizontal distances to create a line segment with the given slope. From points *A* to *G*: $\frac{3}{4}$. From points *B* to *H*: $\frac{6}{8}$. From points *C* to *J*: $\frac{-3}{-4}$.



2. What do you notice about these line segments?

3. For each point, count vertical and horizontal distances to create a line segment with the given slope. From points *D* to *K*: $\frac{6}{-3}$. From points *E* to L: -2. From points *F* to *M*: $\frac{-4}{2}$.



4. What do you notice about these line segments?

REVIEW: INTEGER SUBTRACTION AND DIVISION

Evaluate each pair of expressions.

1. $6 - 4$ $6 + (-4)$	2. $2 - 3$ $2 + (-3)$	3. $4 - (-1)$ $4 + 1$

4. Complete this statement. It expresses the subtraction rule. $a - b = a + (\_\_\_\_\_)$

Evaluate each pair of expressions.

5. $\frac{10}{5}$ $\frac{-10}{-5}$	6. $\frac{-10}{5}$ $\frac{10}{-5}$

7. Why are both expressions in problem 5 equivalent?

8. Why are both expressions in problem 6 equivalent?

9. Circle the expression below that is equivalent to $\frac{(6)-(4)}{(2)-(3)}$.

$\frac{(4)-(6)}{(3)-(2)}$ $\frac{(6)-(4)}{(3)-(2)}$ $\frac{(4)-(6)}{(2)-(3)}$

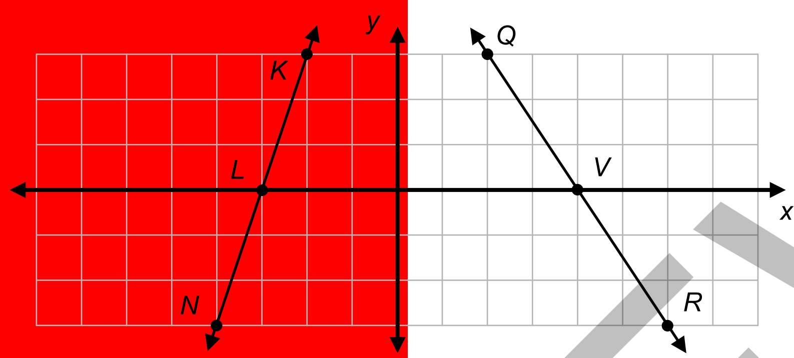
10. Circle the expression below that is equivalent to $\frac{(6)-(4)}{(4)-(-1)}$.

$\frac{(4)-(6)}{(4)-(-1)}$ $\frac{(6)-(4)}{(-1)-(4)}$ $\frac{(4)-(6)}{(-1)-(4)}$

11. In general, if $c \neq d$, then $\frac{a-b}{c-d} = \frac{b-a}{d-c}$. Explain what this means in your own words. You may use expressions from above to help you.

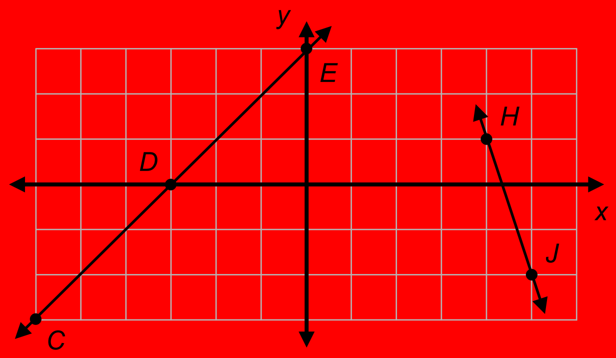
USING COORDINATES TO FIND SLOPE OF A LINE

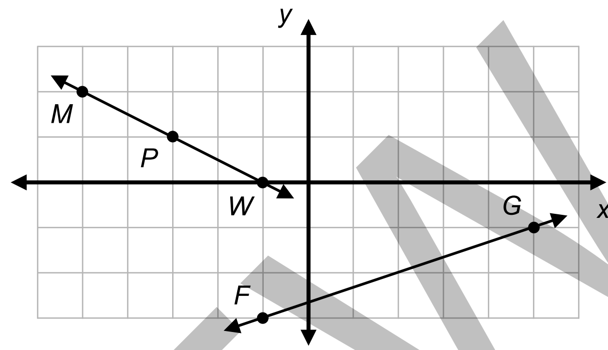
Follow your teacher's directions.



1. Write a definition for slope of a line that involves coordinates. Refer to section 2.5 for help.	
2. + –	3. + –
4. + –	5. + –
6.	

PRACTICE 3





Before calculating, determine whether each slope is positive or negative and circle **+** or **–**. Then use the slope formula to calculate the slope of each line segment. Check by counting.

1. Slope of $\overline{HJ}$: **+** **–**	2. Slope of $\overline{FG}$: **+** **–**
3. Slope of $\overline{CD}$: **+** **–**	4. Slope of $\overline{MW}$: **+** **–**
5. Slope of $\overline{EC}$: **+** **–**	6. Slope of $\overline{PM}$: **+** **–**

5. Find the slope of the line passing through points *A* and *B* below. Think about whether you will choose to graph the points and find the slope by counting, or use the slope formula.

   A (50 , 100)
   B (20 , 40)

6. In your own words, explain how you can use any two coordinate pairs on a line to calculate the slope of the line.

PRACTICE 4

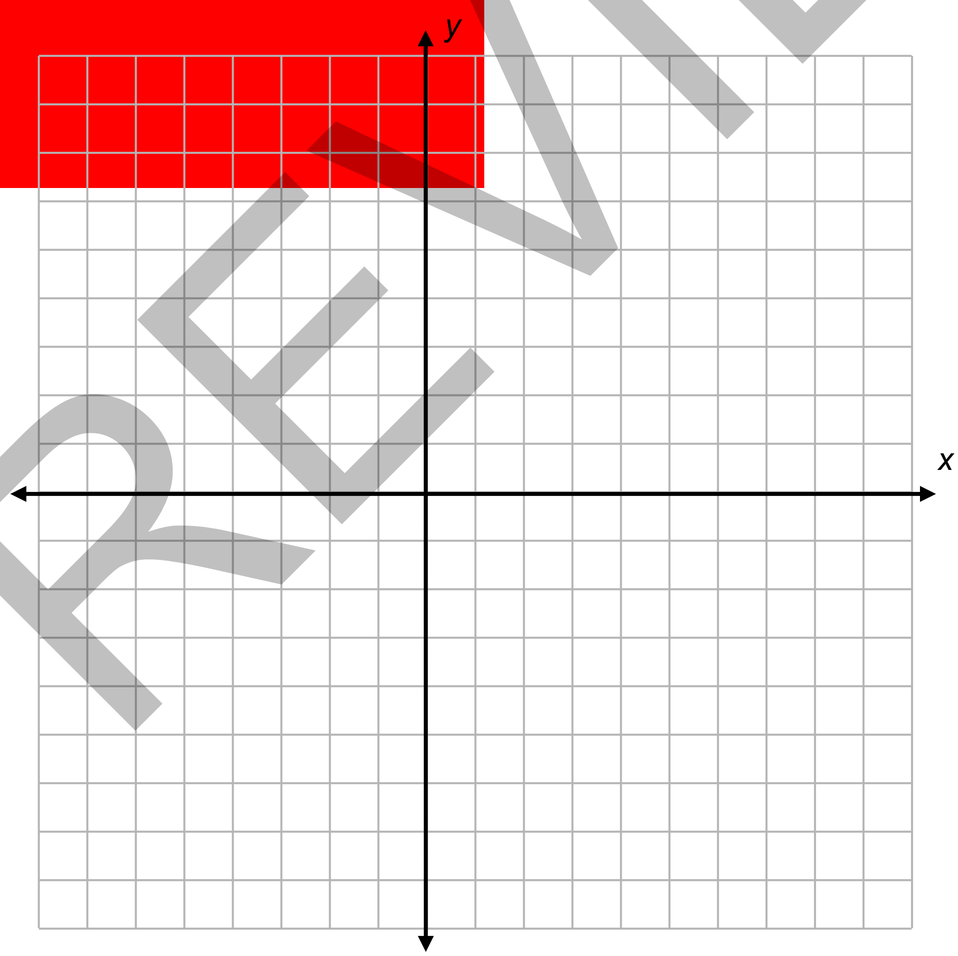
Label each given point and draw the line through it with the given slope.

1. Draw a line through the point *B* (-2, 0) with a slope of $\frac{3}{4}$.
   A point on this line in quadrant I is _____.

2. Draw a line through a point *A* (2, 7) with a slope of $\frac{1}{3}$.
   A point on this line in quadrant II is _____.

3. Draw a line through a point *K* (0, -6) with a slope of $-\frac{1}{2}$.
   A point on this line in quadrant III is _____.

4. Draw a line through a point *L*(5, 4) with a slope of -4.
   A point on this line in quadrant IV is _____.



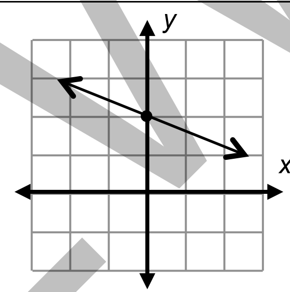
THE *Y*-INTERCEPT

We will use tables of numbers, graphs, and equations to learn about the *y*-intercept of a line.

GETTING STARTED

1. Look in section 2.5 for <u>*y*-intercept</u> and write the meaning of it in My Word Bank.

In the figure to the right, the *y*-intercept is ______ (a single number), and it is located on the *y*-axis with coordinates (______,______)

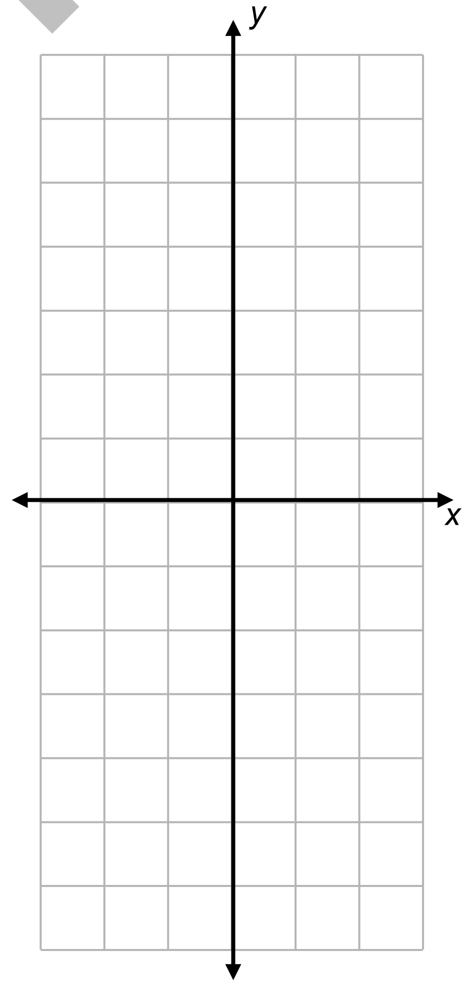


For each function rule, complete the table, graph the line, and identify the slope and *y*-intercept.

2. $y = 2x$	
x	y
0	
1	
2	
-1	
slope:	
y-intercept:	

3. $y = 2x + 4$	
x	y
0	
1	
2	
-1	
slope:	
y-intercept:	

4. $y = 2x - 3$	
x	y
0	
1	
2	
-1	
slope:	
y-intercept:	



5. Explain how to find the *y*-intercept of a line from looking at each of the following.
   a. A graph.
   b. A table.
   c. An equation.

MATCHING ACTIVITY: LINEAR FUNCTION REPRESENTATIONS

1. Your teacher will give you some cards. Match the equations, tables, and graphs.

2. Fill in the missing information.

A. Equation: $y = 2x + 4$ Table match: Graph match: Slope: y-intercept:	B. Equation: $y = 2x$ Table match: Graph match: Slope: y-intercept:	C. Equation: $y = -2x + 4$ Table match: Graph match: Slope: y-intercept:
D. Equation: $y = \frac{1}{2}x$ Table match: Graph match: Slope: y-intercept:	E. Equation: $y = \frac{1}{2}x + 4$ Table match: Graph match: Slope: y-intercept:	F. Equation: $y = -\frac{1}{2}x$ Table match: Graph match: Slope: y-intercept:

3. Circle all the equations below whose graphs are lines parallel to $y = -5x + 4$.

$y = -5x + 1$ $y = -5x - 1$ $y = 5x + 4$ $y = 5x$ $y = -5x$

4. Circle all the equations below whose graphs have the same y-intercept as $y = -5x + 4$.

$y = -2x + 4$ $y = -x - 4$ $y = 5x + 4$ $y = x - (-4)$ $y = -5x$

5. Picture a line that goes through the origin. What is its y-intercept?

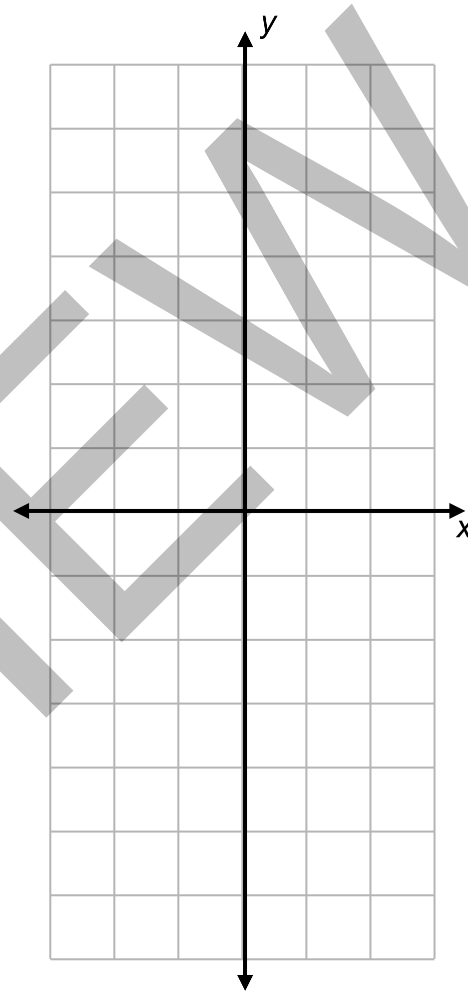
PRACTICE 5

For each function, complete the table, graph the line, and identify the slope and *y*-intercept.

1. $y = 4x$	
x	y
0	
1	
2	
-1	
slope:	
y-intercept:	

2. $y = 4x + 4$	
x	y
0	
1	
2	
-1	
slope:	
y-intercept:	

3. $y = 4x - 3$	
x	y
0	
1	
2	
-1	
slope:	
y-intercept:	



Compare the three equations and graphs in Getting Started to the ones on this page.

4. How are three equations similar?

5. How are the three graphs similar?

For problems 6-8, without making a table or graphing the equations, complete each sentence.

6. For the function rule $y = 5x + 2$, the slope of this line is _____ and the *y*-intercept is _____.

7. For the function rule $y = x - 4$, the slope of this line is _____ and the *y*-intercept is _____.

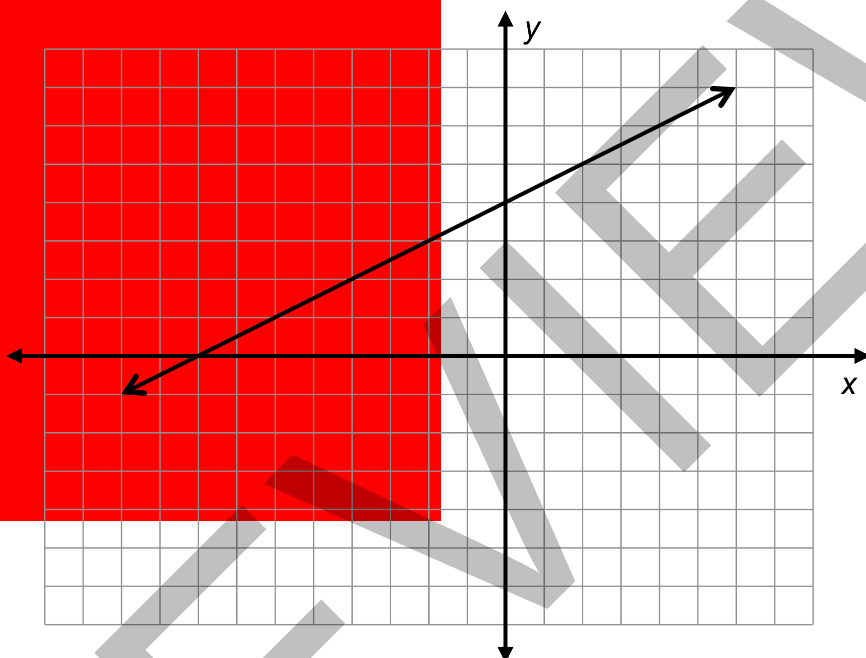
8. For the function rule $y = 3x$, the slope of this line is _____ and the *y*-intercept is _____.

9. What can you say about two or more lines with the same slope?

SLOPE-INTERCEPT FORM

We will find equations of lines in slope-intercept form. We will extend the meaning of slope to horizontal and vertical lines.

GETTING STARTED



Refer to the graph above.

1. When $x = 0$, the y-coordinate is _____.

   This is called the __________.

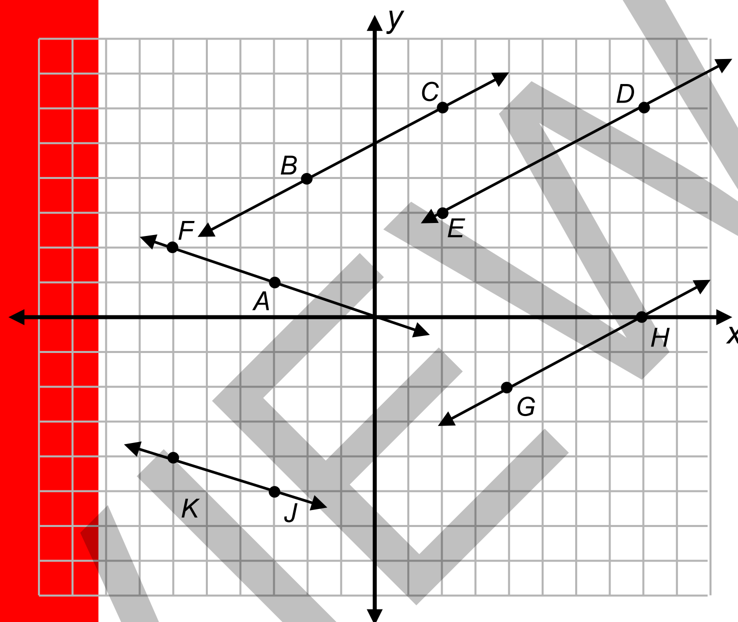
2. Select two points on the line. Find the $\frac{\text{vertical change}}{\text{horizontal change}}$ as you move from one point to another.

   This value is called the __________ of the line.

FINDING EQUATIONS OF LINES

Follow your teacher's directions for 1 and 2.

Slope-intercept form of a line:
1.



2. For line _____

   Slope:

   y-intercept:

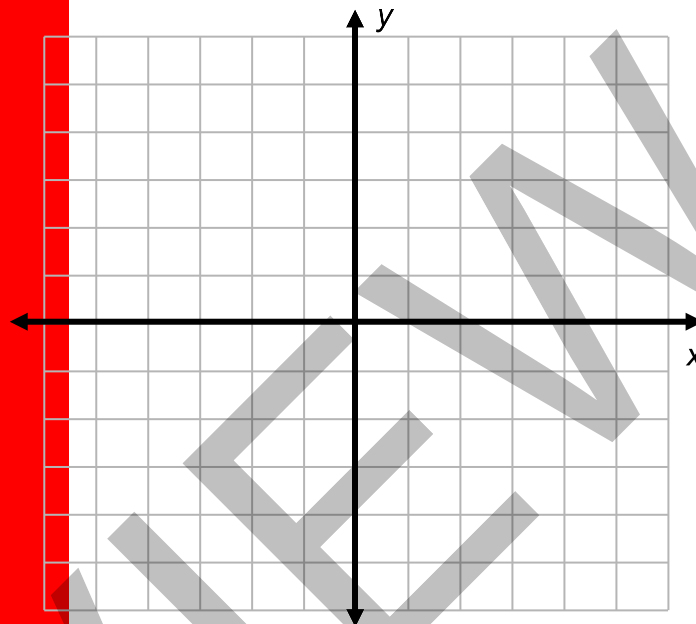
   Equation:

Find the slope, the *y*-intercept, and the equation in slope-intercept form for these lines.

3. Line $\overleftrightarrow{BC}$ Slope: *y*-intercept: Equation:	4. Line $\overleftrightarrow{FA}$ Slope: *y*-intercept: Equation:
5. Line $\overleftrightarrow{JK}$ Slope: *y*-intercept: Equation:	6. Line $\overleftrightarrow{HG}$ Slope: *y*-intercept: Equation:

PRACTICE 6

1. Graph the line that goes through the origin and the point (5, 6).



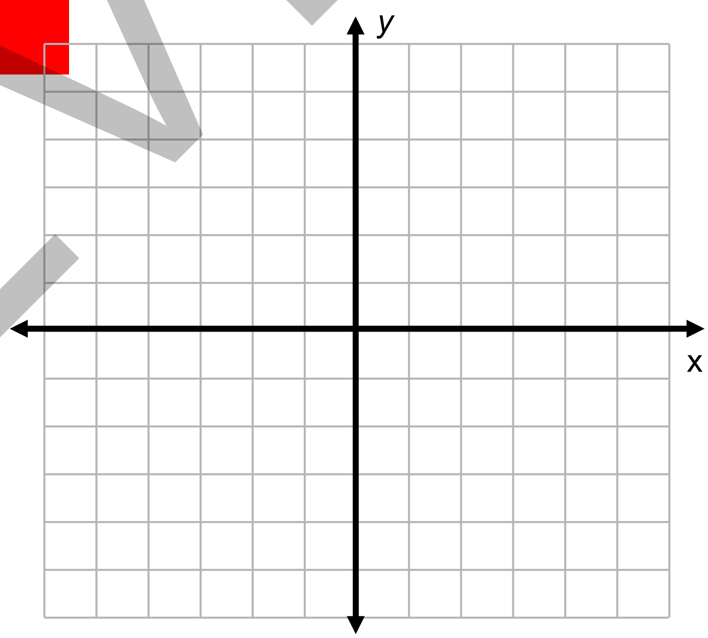
Slope:

y-intercept:

Equation:

Use your equation to determine that the point (-5,-6) lies on the line.

2. Graph the line that goes through (-1, 2) and has a slope of 2.



Slope:

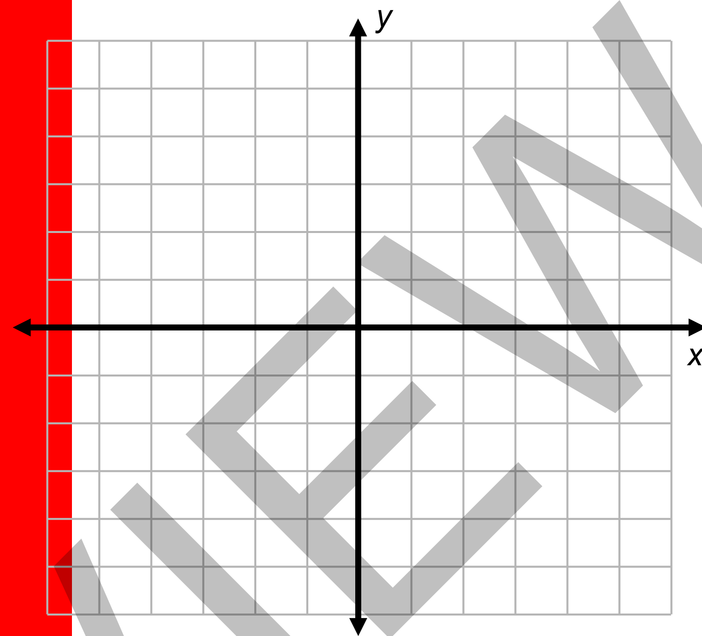
y-intercept:

Equation:

Use your equation to determine that the point (1, 5) does NOT lie on the line.

PRACTICE 6
(Continued)

3. Graph the line that goes through the points (2, 1) and (-2, 3).



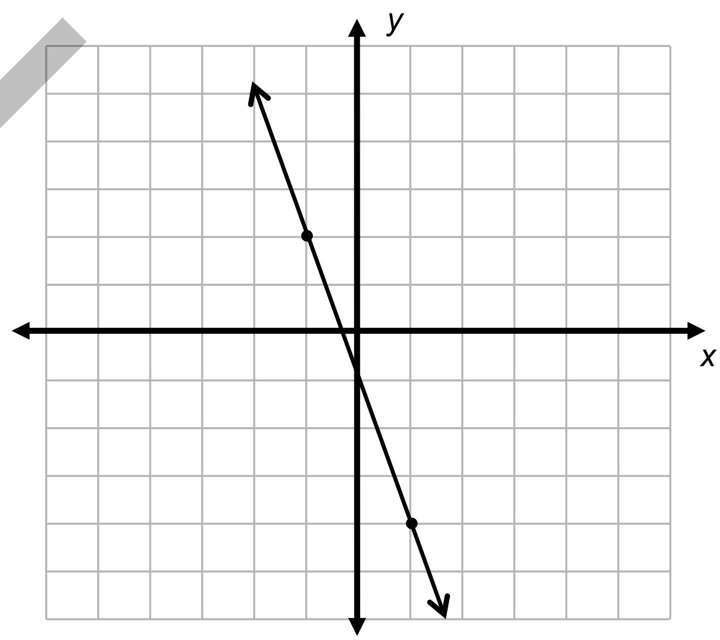
Slope:

y-intercept:

Equation:

Use your equation to determine whether the point (8, -2) lies on the line or not.

4. Write the following for the line graphed here.



Slope:

y-intercept:

Equation:

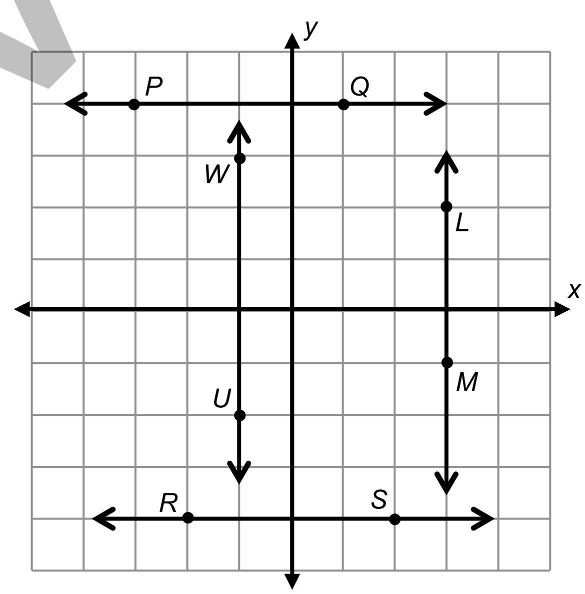
Use your equation to show whether the point (2, -6) lies on the line or not.

HORIZONTAL AND VERTICAL LINES

Fill in the table below to explore properties of equations of horizontal and vertical lines. First circle H if the line is horizontal or V if it is vertical.

Line	Two points on the line	Slope calculation	*y*-intercept (if it exists)	equation
1. $\overleftrightarrow{PQ}$ H V	*Q* (______) *P* (______)			*y* = ______
2. $\overleftrightarrow{WU}$ H V	*U* (______) *W* (______)			*x* = ______
3. $\overleftrightarrow{LM}$ H V	*M* (______) *L* (______)			
4. $\overleftrightarrow{RS}$ H V	*S* (______) *R* (______)			

5. Explain why a *y*-intercept does not exist for the vertical lines to the right.



Recall that the slope-intercept form of a line is

$$y = mx + b$$

6. What is the slope of a horizontal line?

   Is it possible to write the equation of a horizontal line in slope-intercept form?

7. What is the slope of a vertical line?

   Is it possible to write the equation of a vertical line in slope-intercept form?

REVIEW

THE ROPE PROBLEM REVISITED

POSTER PROBLEM: SLOPE AND INTERCEPT

Part 1: Your teacher will divide you into groups.

- Identify members of your group as A, B, C, or D.
- Each group will start at a numbered poster. Our group start poster is _______.
- Each group will have a different colored marker. Our group marker is _________.

Part 2: Do the problems on the posters by following your teacher's directions.

Poster 1 (or 5)	Poster 2 (or 6)	Poster 3 (or 7)	Poster 4 (or 8)
$y = 3x + 1$	$y = 3x - 1$	$y = -3x + 1$	$y = \frac{1}{3}x$

A. Copy the equation. Make a table. For the *x*-values, choose 0, one negative number, and two positive numbers.

B. Graph the line. Scale the *x*-axis and *y*-axis as needed.

C. By looking at the equation, the table, and the graph, write the slope and the *y*-intercept.

D. Double check the slope by choosing two points on the line and calculating it using the slope formula.

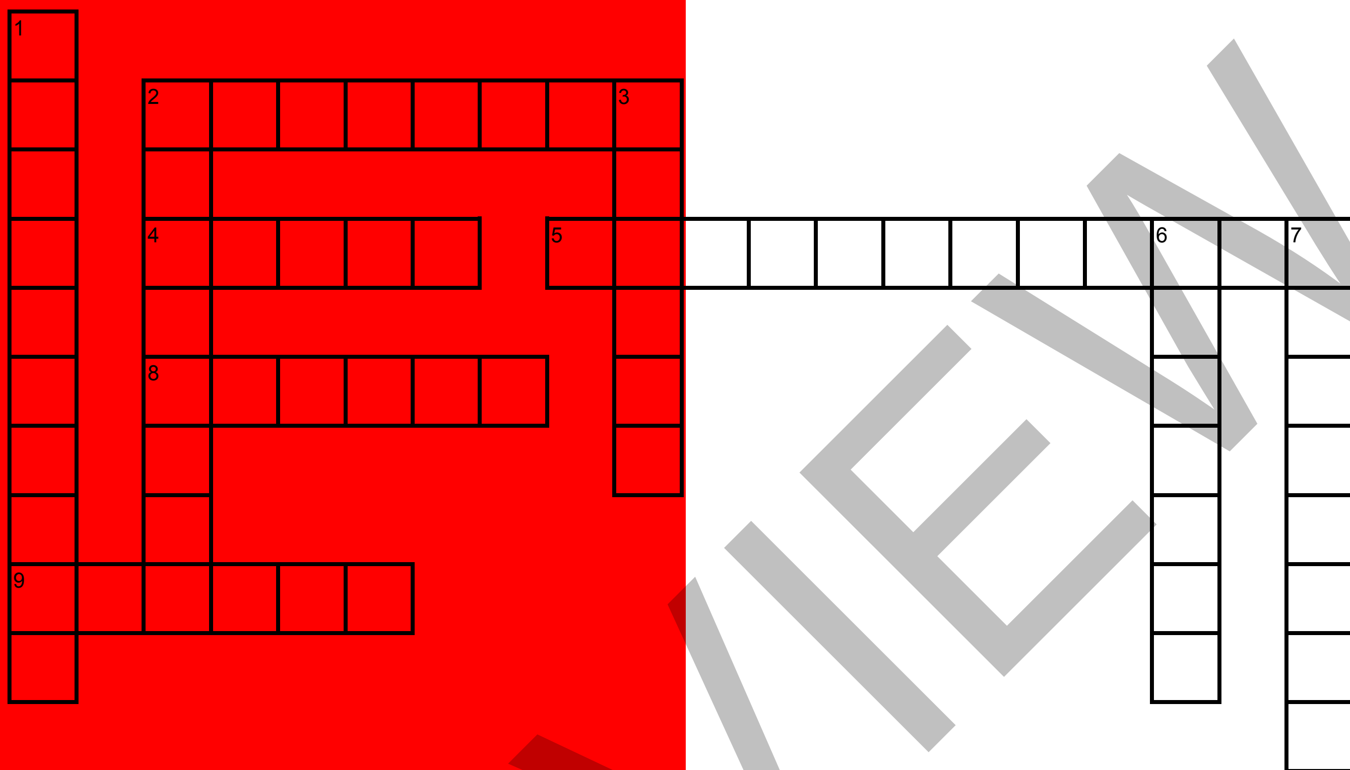
Part 3: Return to your seats. Work with your group.

Use your "start problem" and do the following. Be prepared to share answers with the class.

1. Write two different equations that have the same slope as your equation but different *y*-intercepts. Explain how you know you are correct.

2. Write two different equations that have the same *y*-intercept as your equation but different slopes. Explain how you know you are correct.

VOCABULARY REVIEW



Across

2 Lines that never meet.

4 Steepness of a line.

5 Where lines cross; point of ______.

8 The slope of the line $y = 12x + 20$ (in words).

9 The y-intercept of the line $y = 12x + 20$ (in words).

Down

1 For the ordered pair (3, 9), the y-______ is 9.

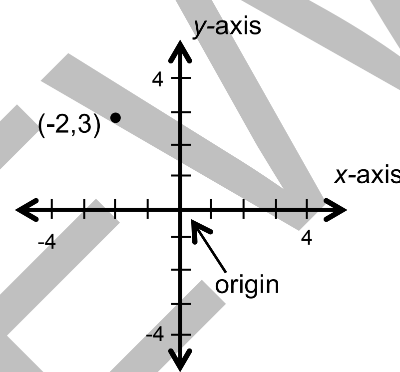
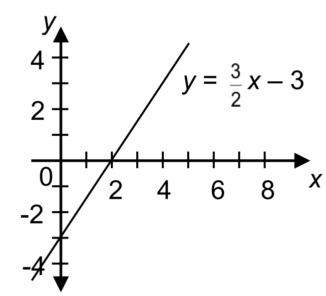
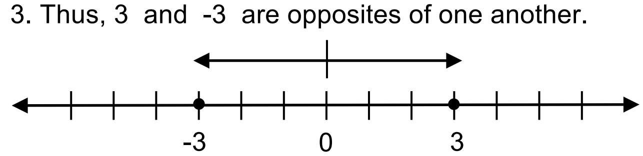
2 Description of the slope of a line that goes through the 1st and 3rd quadrants.

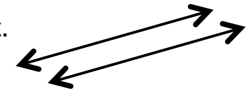
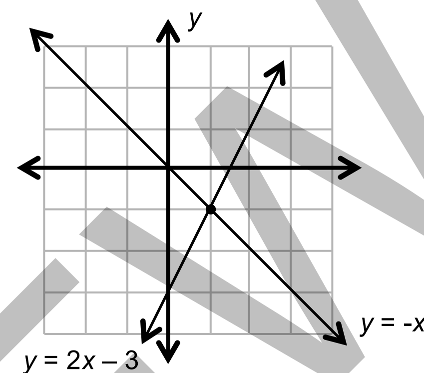
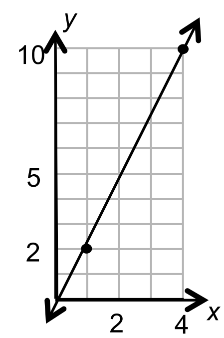
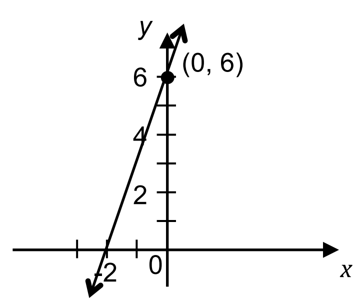
3 A function whose graph is a straight line.

6 The opposite of a number is its additive ______.

7 Description of the slope of a line that goes through the 2nd and 4th quadrants.

DEFINITIONS, EXPLANATIONS, AND EXAMPLES

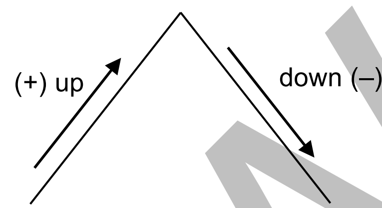
Word or Phrase	Definition
coordinate plane	A <u>coordinate plane</u> is a plane with two perpendicular number lines (<u>coordinate axes</u>) meeting at a point (the <u>origin</u>). Each point *P* of the coordinate plane corresponds to an ordered pair (*a*, *b*) of numbers, called the <u>coordinates</u> of *P*. The coordinate axes are often referred to as the <u>*x*-axis</u> and the <u>*y*-axis</u> respectively. Points on the *x*-axis have coordinates (*a*, 0), and points on the *y*-axis have coordinates (0, *b*). The origin has coordinates (0, 0). 
linear function	A <u>linear function</u> (in variables *x* and *y*) is a function that can be expressed in the form $y = mx + b$. The graph of $y = mx + b$ is a straight line with slope *m* and *y*-intercept *b*. The graph of the linear function $y = \frac{3}{2}x - 3$ is a straight line with slope $m = \frac{3}{2}$ and *y*-intercept $b = -3$. 
opposite of a number	The <u>opposite of a number</u> *n*, written -*n*, is its additive inverse. Algebraically, the sum of a number and its opposite is zero. Geometrically, the opposite of a number is its reflection through zero on the number line. The opposite of 3 is -3, because 3 + (-3) = -3 + 3 = 0. Similarly, the opposite of -3 is -(-3) = 3. Thus, 3 and -3 are opposites of one another.  **Heads Up!** The opposite of a number does NOT have to be negative. Also, the opposite of a number is not necessarily its absolute value.

Word or Phrase	Definition
parallel	Two lines in a plane are <u>parallel</u> if they do not meet. 
point of intersection	A <u>point of intersection</u> of two lines is a point where the lines meet. The two straight lines in the plane with equations $y = -x$ and $y = 2x - 3$ have point of intersection (1, -1). 
slope-intercept form	The <u>slope-intercept form</u> of the equation of a line is the equation $y = mx + b$, where m is the slope of the line, and b is the y-intercept of the line. The equation $y = 2x + 3$ determines a line with slope 2 and y-intercept 3.
slope of a line	The <u>slope of a line</u> is the vertical change (change in the y-value) per unit of horizontal change (change in the x-value). The slope of the line through (1, 2) and (4, 10) is $\frac{8}{3}$: $\text{slope} = \frac{(\text{difference in } y)}{(\text{difference in } x)} = \frac{(10-2)}{(4-1)} = \frac{8}{3}$ 
y-intercept	The <u>y-intercept</u> of a line is the y-coordinate of the point at which the line crosses the y-axis. It is the value of y that corresponds to $x = 0$. For the line $y = 3x + 6$, the y-intercept is 6. If $x = 0$, then $y = 6$. 

Slope

Roughly speaking, slope (m) is the "slant" of a line.

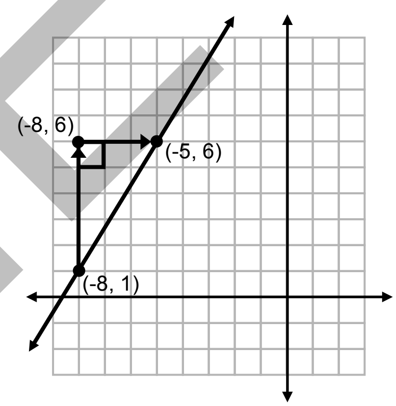
One way to think about whether slope is positive or negative is to imagine that the line is a portion of a mountain (m). Just as we read from left to right, we will move up and down the mountain from left to right. When moving up the mountain, the slope is positive. When moving down the mountain, the slope is negative. The steeper the mountain, the greater (in absolute value) the slope.



The slope (m) of a line is computed as: $\dfrac{\text{vertical change}}{\text{horizontal change}}$ as you move from one point to another on the same line, or $\dfrac{\text{difference in } y \text{ coordinates}}{\text{difference in } x \text{ coordinates}}$ as you move from one point to another on the same line.

To use counting to determine slope, first move in a vertical direction and find the directed distance, and then move in a horizontal direction and find the directed distance.

If A (-8, 1) and B (-5, 6) are points on a line, then count 5 units up and then 3 units to the right. $m = \dfrac{5}{3}$



To use coordinates to determine slope (m), find the quotient of the difference in the y-coordinates and the difference in the x-coordinates.

If A (-8, 1) and B (-5, 6) are points on a line, then

$$m = \frac{\text{difference in } y}{\text{difference in } x} = \frac{6-1}{-5-(-8)} = \frac{5}{3}.$$

If (a, b) and (c, d) are points on a line, then

$$m = \frac{\text{difference in } y}{\text{difference in } x} = \frac{d-b}{c-a}.$$ This formula is the definition of the slope of a line.

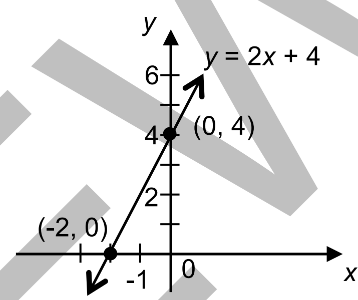
The Slope-Intercept Form of Linear Equations

Slope-intercept form of a linear equation is $y = mx + b$, where m = slope of the line and b = the y-intercept.

Find the equation of a line with a slope of $-\frac{1}{3}$ and the y-intercept is -5.

Since $y = mx + b$, then $y = -\frac{1}{3}x - 5$.

Find the equation of the line that passes through the points (0, 4) and (-2, 0).



First plot the points on a graph.

Notice that the y-intercept is 4.

Count or compute to find the slope,

$$m = \frac{4-0}{0-(-2)} = 2$$

Therefore, the equation of the line is $y = 2x + 4$.

The Slope of Horizontal and Vertical Lines

- The slope of a horizontal line is zero.

A horizontal line is a line parallel to the x-axis. Every point on a horizontal line has the same y-coordinate, and the vertical change between any two positions on the line is zero. Hence,

horizontal

$$\text{slope} = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{0}{\text{horizontal change}} = 0.$$

- The slope of a vertical line is undefined.

A vertical line is a line parallel to the y-axis. Every point on a vertical line has the same x-coordinate, and the horizontal change between any two points on the line is zero. Hence,

vertical

$$\text{slope} = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{\text{vertical change}}{0}$$ is undefined, since division by zero is undefined.

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